Haverhill High School
Haverhill, Massachusetts

AP Calculus BC

Syllabus

Number crunching and symbol manipulation are only small parts of learning calculus. It is also important that students feel they understand each concept studied. One of the major goals is for students to learn how to use precise language to describe these concepts and the relationships between ideas. With the calculus we will analyze functions graphically, numerically, analytically and verbally.

Course Planner

Primary Textbook
Finney, Ross L., et al. *Calculus: Graphical, Numerical, Algebraic*. Menlo Park, Calif.: Scott Foresman/Addison Wesley, 1999.

Chapter 1: Prerequisites for Calculus (7 days)

- Elementary functions
  - Linear, power, exponential/logarithmic, trigonometric/inverse trigonometric
- Parametric equations
- Getting familiar with the graphing calculator

Chapter 2: Limits and Continuity (10 days)

- Limits:
  - Limit at a point, limit at infinity, infinite limits
  - Properties of limits
- Continuity
- Tangent line to a curve
- Slope of a curve at a point

Chapter 3: Derivatives (15 days)

- Definition of f'
- Derivative at a point
- Relating the graphs of f and f'
- When does $f'(a)$ fail to exist?
- Rules for differentiation:
  - Sum, product, quotient
- Chain Rule
- Implicit Differentiation
- Derivatives of trigonometric, inverse trigonometric, exponential and logarithmic functions

Chapter 4: Applications of Derivatives (17 days)

- Mean Value Theorem
- Using the derivative to find:
  - Critical point(s) and extreme values
  - When the function is increasing or decreasing
  - Point(s) of inflection
  - When the function is concave up or concave down
- Optimization problems
- Using the tangent line to approximate function values
- Newton's Method
- Differentials and change
- Related rates

Chapter 5: The Definite Integral (14 days)

- RAM (Rectangle Approximation Method)
- Riemann Sums
- Finding an antiderivative
- Using a definite integral to find area, volume, average value of a function
- Fundamental Theorem of Calculus
- Approximating the definite integral:
  - Trapezoidal Rule, Simpson's Rule, Error Analysis

Chapter 6: Differential Equations and Mathematical Modeling (15 days)

- Slope fields
- Antiderivatives and the indefinite integral
- Techniques of integration
  - Substitution, integration by parts, trigonometric substitution, partial fractions
- Separable differential equations
- Euler's Method
- Exponential growth and decay
- Logistic growth

Chapter 7: Applications of Definite Integrals (12 days)

- Using the definite integral to discuss:
  - Net change—motion on a line, consumption over time
  - Area, volume, length of a curve, surface are of a solid of revolution
  - Work, fluid force

Chapter 8: L'Hopital's Rule, Improper Integrals, Partial Fractions (13 days)

- Indeterminate forms $\left(\frac{0}{0}, \frac{\infty}{\infty}, -\infty, 1^{\infty}, 0^{0}, \infty^{0}\right)$ and L'Hopital's Rule
- Relative rates of growth
- Improper integrals (partial fractions and trig substitutions—done with Chapter 6)

Chapter 10: Parametric, Vector, and Polar Functions (14 days)

- Parametric Functions
  - Derivative at a point
  - $\dfrac{d^2y}{dx^2}$
  - Length of a curve, surface area of a solid of revolution

- Vectors:
  - Angle between vectors
  - Scalar product
  - Using vectors to describe motion in a plane

- Polar Coordinates and pole graphs:
  - Slope, horizontal and vertical tangent lines
  - Area, length of a curve

Chapter 9: Infinite Series (15 days)

- Geometric series
- Power series
  - Term-by-term differentiation and integration to find power series of new functions
- Taylor series/Maclaurin series
- Lagrange form of the remainder
- Tests for convergence/divergence
  - nth term test
  - Direct Comparison
  - Ratio test
  - Integral test
  - Limit comparison test
  - Alternating series test (Leibniz's Theorem)

- Radius and interval of convergence

Review for AP Exam

We go through a few free-response questions in class and discuss the scoring guidelines so students understand the need for "complete" solutions. Over the next two weeks, students work on free-response sections from three previous AP Calculus Exams (all six questions in each exam) at home. They usually have two nights or a weekend to do each test.

Students are encouraged to talk to each other but supposed to write their own solutions. They also take a couple of free-response "minitests" in class. I choose four of the six questions (two from the calculator section and two from the non-calculator section), and they work on these under AP Exam-like conditions—that is, 20 minutes with the calculator for the first two questions, and the remainder of the 42-minute period without the calculator to finish the test.

Teaching Strategies

AP Calculus Course Description

The topic outline shown above is the skeleton of our course. We study every area mentioned in the *AP Calculus Course Description.* We also include some other topics—Newton's Method, Simpson's Rule, integration by trigonometric substitution, volume by cylindrical shells, work—to provide alternate methods of solution or to give an additional look at how calculus zooms into a function to inspect what might be happening at a particular instant or puts many small pieces of information together to say something abut the whole function.

Functions from Multiple Representations

Functions represented analytically comprise one segment of our study of calculus. There are so many situations that use functions described verbally, graphically, or numerically, and we work with those along the way.

When a function is presented graphically we can describe its behavior in broad strokes, and sometimes we can describe the details of its behavior at some specific points. On the other hand, when a function described analytically is graphed, we are careful to check that technological "features" do not mask some behavior.

We work very early in the year with functions described numerically, but at that time we use the regression equations given by TI-83's STAT menu. As we learn more calculus, we use that data (without a regression equation) to approximate the rate of change at a particular time or the accumulated change over a specified period of time. Students need constant remainders that the regression formula is *not* necessarily the analytic description of the function described by the data.

Functions described verbally have been part of the course for a long time. Verbal descriptions about the rate of change of a function are often used to create differential equations, and we solve those equations, either analytically or numerically.

Graphing Calculator

The students come to BC Calculus already knowing the basics of operating a graphing calculator. They can graph a function in a variable-sized window, and they know how to "read" that graph. The first chapter of our textbook reviews the various elementary functions with examples from the real world, so this is a good time to introduce regression equations that the STAT menu gives us. We use these regression equations to make projections about data and to address the very important question about the reasonableness of the projection. The TABLE feature builds on the graph in our initial discussions of the limit of a function. Our slope field program provides a view of a family of antiderivatives, especially those that cannot be found analytically. Many calculator-dependent solutions are based on manipulating graphs or on understanding how several graphs are related to each other. Students adapt quickly to using their calculators in clever ways, experimenting to find possible answers and solutions to problems, even some problems that are easily solved analytically.

The instruction “support graphically” has become an often-used tool since it is so quick to do and gives students confidence in their conjectures and answers. Some students are quick to say, “my calculator tells me so,” and we try to turn the question around to “and what do you think about that?” or “why does the calculator give that result?”

My goal for each class period is to develop an interesting discussion. I tend to get into the lecturer mode too easily, so I always welcome question or comments. In fact, some of the best classes occur when a student asks “Why?” or “Why not?” and the class proposes answers that need to be examined.

Students discuss homework questions in small groups (two to four members) for a few minutes, and unresolved questions are saved for discussion with the entire class. Individual students serve for several days or several weeks as the “homework boss”. In that role, the student leads the homework review for the entire class and solicits volunteers who share their work. The “homework boss” also ensures that everyone else is following along.

Student Evaluation

Students meet in small groups of two to fur for a few minutes at the beginning of the class period to discuss immediate questions from the assignment. Questions that are not resolved are passed on for discussion with the entire class. Quizzes are unannounced and cover small pieces of material, usually work from the prior two or three days. Test cover larger amounts of material and are almost always cumulative. In-class test are divided into calculator/non-calculator sections similar to the free-response section of the AP Exam. I occasionally include some multiple-choice questions, but most of the questions require students to show the work that leads to their answers or are short answer-type questions.

There is one take-home test each of the first three quarters. Students are encouraged to talk about their ideas with their classmates but are told they should write their own solutions to turn in. Tests are not scheduled at the conclusion of chapters or units, but rather when they fit into the schedule. The tests tend to include material that is covered up to a few days before each test is given.